Written Exam at the Department of Economics winter 2016-17

Micro III

Final Exam

Date: 3 January 2017

(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam question consists of 3 pages in total

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

PLEASE ANSWER ALL QUESTIONS.
PLEASE EXPLAIN YOUR ANSWERS.

1. (a) Denote the normal-form game below by G. Solve G by iterated elimination of strictly dominated strategies. Explain briefly each step (1 sentence).

Player 2

		t_1	t_2	t_3
Player 1	s_1	$4,8$	$6,10$	$7,6$
	s_2	$8,4$	$4,2$	$8,0$
	s_3	$6,7$	$12,2$	$5,4$
	s_4	$2,8$	$9,9$	$4,10$

(b) Suppose we repeat the stage game G twice. Denote the resulting game by $G(2)$. Find the set of pure-strategy Subgame-perfect Nash Equilibria of $G(2)$. Be careful to write out the equilibrium strategies.

(c) Consider now the infinitely repeated game with discount factor $\delta < 1$. Denote this game by $G(\delta)$. Is it possible to find a Subgame-perfect Nash Equilibrium of $G(\delta)$, for at least some values of $\delta < 1$, in which the average payoff of both players is strictly higher than their payoff in every Nash Equilibrium of G (i.e. of the 1-period game seen in part (a))? If so, find such an equilibrium. If not, argue why it does not exist. If you found such an equilibrium, be careful to argue why it is subgame perfect, and to show that neither player has an incentive to deviate from his equilibrium strategy.

2. Consider the extensive-form game given by the following game tree (the first payoff is that of player 1, the second payoff that of player 2):

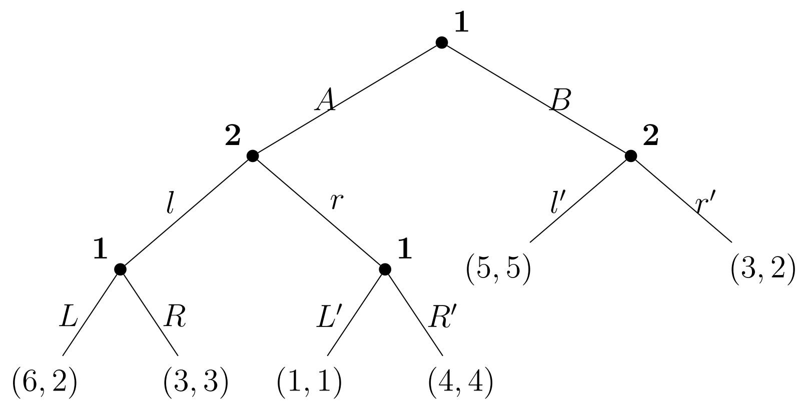

FIGURE 1

(a) How many proper subgames are there (excluding the game itself)? What are the strategy sets of the players?

(b) Find all (pure strategy) Subgame-perfect Nash Equilibria.

(c) Suppose now that player 1 does not observe the move of player 2, in situations where player 1 is called upon to move for a second time. That is to say, if player 1 chooses A, he does not observe whether player 2 then chooses l or r.

   i. Draw the resulting game tree.

   ii. Is this a game of perfect or imperfect information? How many proper subgames are there (excluding the game itself)? What are the strategy sets of the players?

   iii. Show that there is a Subgame-Perfect Nash Equilibrium where player 1 has payoff 6. Briefly discuss why player 1 benefits from not being able to observe player 2's action (max. 3 sentences).

3. Two tech entrepreneurs have made a gazillion dollar through a new app and need to decide how to allocate the gains. If they can't agree, nobody gets anything. Let x_1 and x_2 be the amounts that entrepreneur 1 and 2 get. Then their utilities are:

$$\begin{aligned} u_1(x_1) &= x_1 \\ u_2(x_2) &= 2\sqrt{x_2}. \end{aligned}$$

Find the Nash Bargaining Solution. What are the allocations?

4. Suppose we are in a setting similar to the *common value auction* seen in class. There are two bidders, $i = 1, 2$. A single object is being sold in a **first-price auction**. Each bidder receives a uniformly distributed signal: for $i = 1, 2$,

$$s_i \sim u(0, 1).$$

Suppose that s_1 and s_2 are independent. The bidders' valuations depend on *both* signals, and are given by

$$\begin{aligned} v_1 &= \frac{3}{4}s_1 + \frac{1}{4}s_2, \\ v_2 &= \frac{3}{4}s_2 + \frac{1}{4}s_1. \end{aligned}$$

Thus, the valuation of each bidder is 3/4 times his own signal plus 1/4 times his competitor's signal.

   (a) What is the expected value of the object to the bidders *before* they enter the auction?

   (b) Suppose the two bidders follow a symmetric linear strategy $\beta_i(s_i) = as_i$, where $a > 0$ is a positive constant. Conditional on s_1 and on winning the auction, what is player 1's expected value of the object? Explain why this is different to your answer in (a).

   (c) Show that there is an equilibrium with strategies of the form $\beta_i(\cdot)$, as seen in part (b). Explicitly solve for these equilibrium strategies i.e. find a.